Claims

What is claimed is:

5   1. A method of partitioning the columns of a matrix A, said method comprising:

providing the matrix A in a memory device of a computer system, said matrix A having n

columns and m rows, n being an integer of at least 3, m being an integer of at least 1; and

executing an algorithm by a processor of the computer system, said executing including

partitioning the n columns of the matrix A into a closed group of p clusters, p being a positive

10   integer of at least 2 and less than n, said partitioning comprising an affinity-based merging of

clusters of the matrix A, each said cluster consisting of one or more columns of said matrix A.

2. The method of claim 1, said executing further including accepting p as an input to the

algorithm.

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3. The method of claim 1, wherein the matrix A relates a vector x having n elements to a vector d

having m elements in accordance with an equation of $d = Ax$, wherein the n elements of the

vector x consist of n statistically independent variables, and wherein the m elements of the vector

d consist of m dependent variables.

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4. The method of claim 3, said executing further including after said partitioning:

computing a vector z having p statistically independent elements such that each of the p statistically independent elements is a linear combination of the n statistically independent variables; and

computing an m x p matrix B from the p clusters of the matrix A such that Bz defines a

5   new set of m dependent variables replacing Ax.

5. The method of claim 3, said n statistically independent variables representing non-gaussian sources of variation.

10   6. The method of claim 5, said executing further including selecting the n statistically independent variables from N statistically independent variables such that $N > n$, said N variables consisting of said n variables and a remaining $N - n$ statistically independent variables, said $N - n$ variables representing gaussian sources of variation.

15   7. The method of claim 3, said m elements of the vector d denoting path slack variations in a semiconductor chip, said n statistically independent variables denoting sources of statistical error that linearly contribute to said path slack variations.

8. The method of claim 7, said sources of statistical error comprising statistical variations

20   selected from the group consisting of statistical variations associated with processing the semiconductor chip, statistical variations associated with manufacturing the semiconductor chip, statistical variations associated with operating the semiconductor chip, statistical variations

associated with modeling the semiconductor chip, and statistical variations associated with

uncertainties in material properties of the semiconductor chip.

9. A method of partitioning the columns of a matrix A, said method comprising executing an algorithm by a processor of a computer system, said executing including performing the steps of:

generating a list of clusters having n clusters such that each of the n clusters consist of a unique column of the matrix A, said matrix A being stored in a memory device of the computer system, said matrix A having n columns and m rows, n being an integer of at least 3, m being an integer of at least 1, each said cluster consisting of one or more columns of said matrix A;

determining if a termination condition is satisfied and if said determining so determines that said termination condition is satisfied then terminating said executing else performing the following steps:

selecting a next pair of clusters from the list of clusters, said next pair of clusters consisting of a first cluster and a second cluster, said next pair of clusters having an affinity that is not less than an affinity between any pair of clusters not yet selected from the list of clusters;

merging the first and second clusters to form a new cluster;

inserting the new cluster into the list of clusters while removing the first and second clusters from the list of clusters; and

re-executing said determining step.

10. The method of claim 9, said executing further including performing the step of accepting an input p to the algorithm, p being a positive integer of at least 2 and less than n, said termination condition being that a current number of clusters in the list of clusters is equal to p.

11. The method of claim 9, said executing further including performing the steps of:

accepting an affinity threshold as an input to the algorithm; and

if the affinity of the next pair of clusters selected in the selecting step is less than the

affinity threshold then setting a flag indicating that the termination condition has been satisfied

and again performing the determining step while not performing the inserting step.

12. The method of claim 9, said executing further including performing the steps of:

supplying a cluster error tolerance ϵ as an input to the algorithm; and

if the selecting step results in the list of clusters having a cluster approximation error E

such that $E \geq \epsilon$ then setting a flag indicating that the termination condition has been satisfied and

again performing the determining step while not performing the inserting step.

13. The method of claim 9, wherein the matrix A relates a vector x having n elements to a vector

d having m elements in accordance with an equation of $d = Ax$, wherein the n elements of the

vector x consist of n statistically independent variables, and wherein the m elements of the vector

d consist of m dependent variables.

14. The method of claim 13, said executing further including performing the steps of:

computing a vector z having p statistically independent elements such that each of the p

statistically independent elements is a linear combination of the n statistically independent

variables; and

computing an $m \times p$ matrix B from the p clusters of the matrix A such that Bz defines a new set of m dependent variables replacing Ax.

15. The method of claim 13, said n statistically independent variables representing non-gaussian sources of variation.

16. The method of claim 15, said executing further including performing the step of selecting the n statistically independent variables from N statistically independent variables such that $N > n$, said N variables consisting of said n variables and a remaining $N - n$ statistically independent variables, said $N - n$ variables representing gaussian sources of variation.

17. The method of claim 13, said m elements of the vector d denoting path slack variations in a semiconductor chip, said n statistically independent variables denoting sources of statistical error that linearly contribute to said path slack variations.

18. The method of claim17, said sources of statistical error comprising statistical variations selected from the group consisting of statistical variations associated with processing the semiconductor chip, statistical variations associated with manufacturing the semiconductor chip, statistical variations associated with operating the semiconductor chip, statistical variations associated with modeling the semiconductor chip, and statistical variations associated with uncertainties in material properties of the semiconductor chip.

19. A computer program product, comprising a computer usable medium having a computer readable program embodied therein, said computer readable program comprising an algorithm for partitioning the columns of a matrix A, said algorithm adapted to perform the steps of:

providing the matrix A in a memory device of the computer system, said matrix A having n columns and m rows, n being an integer of at least 3, m being an integer of at least 1; and

partitioning the n columns of the matrix A into a closed group of p clusters, p being a positive integer of at least 2 and less than n, said partitioning comprising an affinity-based merging of clusters of the matrix A, each said cluster consisting of one or more columns of said matrix A.

20. The computer program product of claim 19, wherein the algorithm is adapted to perform the step of accepting p as an input to the algorithm.

21. The computer program product of claim 19, wherein the matrix A relates a vector x having n elements to a vector d having m elements in accordance with an equation of $d = Ax$, wherein the n elements of the vector x consist of n statistically independent variables, and wherein the m elements of the vector d consist of m dependent variables.

22. The computer program product of claim 21, wherein after said partitioning the algorithm is adapted to perform the steps of:

computing a vector z having p statistically independent elements such that each of the p statistically independent elements is a linear combination of the n statistically independent variables; and

computing an $m \times p$ matrix B from the p clusters of the matrix A such that Bz defines a

5   new set of m dependent variables replacing Ax.

23. The computer program product of claim 21, said n statistically independent variables representing non-gaussian sources of variation.

10   24. The computer program product of claim 23, wherein the algorithm is further adapted to perform the step of selecting the n statistically independent variables from N statistically independent variables such that $N > n$, said N variables consisting of said n variables and a remaining $N - n$ statistically independent variables, said $N - n$ variables representing gaussian sources of variation.

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25. The computer program product of claim 21, said m elements of the vector d denoting path slack variations in a semiconductor chip, said n statistically independent variables denoting sources of statistical error that linearly contribute to said path slack variations.

20   26. The computer program product of claim 25, said sources of statistical error comprising statistical variations selected from the group consisting of statistical variations associated with processing the semiconductor chip, statistical variations associated with manufacturing the

semiconductor chip, statistical variations associated with operating the semiconductor chip,

statistical variations associated with modeling the semiconductor chip, and statistical variations

associated with uncertainties in material properties of the semiconductor chip.

27. A computer program product, comprising a computer usable medium having a computer readable program embodied therein, said computer readable program comprising an algorithm for partitioning the columns of a matrix A, said algorithm adapted to perform the steps of:

generating a list of clusters having n clusters such that each of the n clusters is a unique column of the matrix A, said matrix A being stored in a memory device of the computer system, said matrix A having n columns and m rows, n being an integer of at least 2, m being an integer of at least 1, each said cluster consisting of one or more columns of said matrix A;

determining if a termination condition is satisfied and if said determining so determines that said termination condition is satisfied then terminating said algorithm else executing the following steps:

selecting a next pair of clusters from the list of clusters, said next pair of clusters consisting of a first cluster and a second cluster, said next pair of clusters having an affinity that is not less than an affinity between any pair of clusters not yet selected from the list of clusters;

merging the first and second clusters to form a new cluster;

inserting the new cluster into the list of clusters while removing the first and second clusters from the list of clusters; and

re-executing said determining step.

28. The computer program product of claim 27, wherein the algorithm is adapted to accept p as an input to the algorithm, p being a positive integer of at least 2 and less than n, said termination condition being that a current number of clusters in the list of clusters is equal to p.

29. The computer program product of claim 27, wherein the algorithm is adapted to accept an affinity threshold as an input to the algorithm, wherein if the affinity of the next pair of clusters selected in the selecting step is less than the affinity threshold then the algorithm is adapted to execute setting a flag indicating that the termination condition has been satisfied and again performing the determining step while not performing the inserting step.

30. The computer program product of claim 27, wherein the algorithm is adapted to accept a cluster error tolerance ϵ as an input to the algorithm, wherein if the selecting step results in the list of clusters having a cluster approximation error E such that $E \geq \epsilon$ then the algorithm is adapted to execute setting a flag indicating that the termination condition has been satisfied and again performing the determining step while not performing the inserting step.

31. The computer program product of claim 27, wherein the matrix A relates a vector x having n elements to a vector d having m elements in accordance with an equation of $d = Ax$, wherein the n elements of the vector x consist of n statistically independent variables, and wherein the m elements of the vector d consist of m dependent variables.

32. The computer program product of claim 31, said algorithm being adapted to further perform the steps of:

computing a vector z having p statistically independent elements such that each of the p statistically independent elements is a linear combination of the n statistically independent variables; and

computing an m x p matrix B from the p clusters of the matrix A such that Bz defines a

5 new set of m dependent variables replacing Ax.

33. The computer program product of claim 31, said n statistically independent variables representing non-gaussian sources of variation.

10 34. The computer program product of claim 33, said algorithm further adapted to perform the step of selecting the n statistically independent variables from N statistically independent variables such that $N > n$, said N variables consisting of said n variables and a remaining $N - n$ statistically independent variables, said $N - n$ variables representing gaussian sources of variation.

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35. The computer program product of claim 31, said m elements of the vector d denoting path slack variations in a semiconductor chip, said n statistically independent variables denoting sources of statistical error that linearly contribute to said path slack variations.

20 36. The computer program product of claim 35, said sources of statistical error comprising statistical variations selected from the group consisting of statistical variations associated with processing the semiconductor chip, statistical variations associated with manufacturing the

semiconductor chip, statistical variations associated with operating the semiconductor chip,

statistical variations associated with modeling the semiconductor chip, and statistical variations

associated with uncertainties in material properties of the semiconductor chip.

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